Mathematics: applications and interpretation SL

Timezone 1

To protect the integrity of the assessments, increasing use is being made of examination variants. By using variants of the same examination, students in one part of the world will not always be responding to the same examination content as students in other parts of the world. A rigorous process is applied to ensure that the content across all variants is comparable in terms of difficulty and syllabus coverage. In addition, measures are taken during the standardization and grade awarding processes to ensure that the final grade awarded to students is comparable.

Contents

Grade boundaries

Standard level overall

Grade:	1	2	3	4	5	6	7
Mark range:	0–12	13–24	25–37	38–50	51–64	65–76	77–100

Standard level internal assessment

Grade:	1	2	3	4	5	6	7
Mark range:	0–2	3–5	6–8	9–11	12–14	15–17	18–20

Standard level paper one

Grade:	1	2	3	4	5	6	7
Mark range:	0–11	12–19	20–29	30–38	39–49	50–57	58–80

Standard level paper two

Grade:	1	2	3	4	5	6	7
Mark range:	0–10	11–20	21–29	30–40	41–52	53–62	63–80

Standard level internal assessment

The range and suitability of the work submitted

As in previous sessions, most of the explorations were focused on statistics. These explorations were mostly based on the application of Pearson's and/or Spearman's correlation coefficients, χ^2-tests and/or two sample *t*-tests as well as the application of ANOVA. More χ^2-GOF were seen to test for the normality of data distribution. Mathematical modelling was the second prevailing topic. These generally centered around finding functions to fit data using exponential, quadratic, and polynomial functions. There was an increase in the application of Voronoi diagrams. These were generally well done and of good quality. There were less explorations in this exam session on use of calculus than in previous years, most of which incorporated finding the volume of revolution and/or surface area of objects. Other topics noted this year were graph theory and probability. Little to none were seen on finance and geometry. There were less explorations noted where students attempted mathematics outside/above the SL syllabus.

An increase in template like statistical explorations, where the entire sample from a centre looked the same, was noted. Providing templates to students limits their creativity and personal engagement with the mathematics. It also risks an accusation of plagiarism if the work is not sufficiently the student's.

Many explorations consisting of 20 – 30 pages were seen where students included very long introductions, rationales, plans, any and all limitations they could think of and then all the statistical processes in the syllabus losing coherence and organization along the way.

There were a large number of incomplete explorations seen in this session where it appeared that students just did *something* in order to avoid an incomplete final result.

Student performance against each criterion

Criterion A

This criterion showed an overall improvement where most explorations: stated a clear aim; included only processes relevant to the aim; placed large raw data tables in the appendix; clearly introduced new processes; provided explanations and/or discussions for calculations; and placed graphs where appropriate and required. Students who achieved the higher levels in this criterion were those that had a clear aim at the start, stayed focussed, applied only mathematical processes relevant to the aim throughout the exploration and achieved the aim.

Some students still included a plan or a statement of task or a section titled "methodology". These sections often appeared to be summary of the actual exploration, assuming results yet to be determined which negatively affected coherence as well as organization. In many cases the students relied on the plan given in the beginning and thus failed to introduce/link the different processes in a meaningful way to the aim. The processes and methods used should be explained as the exploration develops. If all the processes mentioned in the plan were not completed, then the exploration would no longer be coherent.

Some explorations included an unnecessary "research question" which was sometimes slightly different to the title/aim which caused confusion.

Concision is still an issue. Many explorations included: longwinded introductions trying to establish personal engagement; many pages explaining the processes; and textbook explanations of topics in the syllabus.

On the other hand, some students, in an effort to be the concise, either left out or relegated important, and often vital, calculations and/or data to the appendix affecting coherence and organization.

Students who choose topics outside the syllabus should however ensure that these are explained in sufficient detail for a peer who is only familiar with the Mathematics: A&I SL syllabus to understand. This does not however imply that the student should blindly reproduce information from a textbook or any other source.

Criterion B

The quality of mathematical communication showed some improvement from previous examination sessions. Graph axes were labelled and had appropriate scales in context, variables were explicitly defined in context, the relevance and implication of chosen levels of accuracy was explained/justified, the correct symbol for χ was used, setting out statistical tests using the prescribed format, avoiding calculator notation, and using an equation editor.

Some persistent issues which affected the awarding of higher levels here are the inconsistent use of: upper- and lower-case letters for the same variables; levels of accuracy; italics; function notation such as y as well as $f(x)$; approximate equal symbol.

Some use of *, ^ , "/" and the letter x for multiplication were seen, but less than in previous sessions. Inappropriate and incorrect use of the summation symbol was noted in this session.

There is no need to provide calculator and/or spreadsheet instructions. Screenshots of the GDC are mostly inappropriate as these would include values and results not required and would also be superfluous if the student then proceeded to write the results in the body of the exploration. Variables and formulae should not be presented/mentioned if not used.

Students should be instructed in the correct use and definitions of mathematical terms such as: correlation, prove, test. Null and alternate hypotheses should only be used in the context of statistical tests and not be used as an attempt to indicate personal engagement. Topics such as Voronoi diagrams, have specific terminologies which should be used.

Mathematical calculations and processes should not be described in paragraph form, these processes should be noted in mathematical form.

In an attempt to have multiple forms of mathematical representation, some students presented inappropriate and sometimes irrelevant diagrams and graphs. It should be noted that it clearly states in the Teacher Support Materials (TSM) for this criterion that: "Level 4 can be achieved using only one form of mathematical communication as long as this is appropriate to the topic being explored."

Criterion C

This criterion primarily assesses how engaged the student is in the mathematics of the exploration as clearly stated in the TSM: "Criterion C assesses the extent to which the student engages with the topic by exploring the mathematics and making it their own. It is not a measure of effort."

While the student's own data collection is beneficial, it does not guarantee a level 2 or 3 in criterion C. Students should explore different perspectives, ask questions, and show creative thinking beyond the regular steps seen in textbooks. Enhancing their work with attempts to generalize solutions is advisable.

Learning new maths can certainly be an indication of significant engagement. However, if the new maths is applied in a mechanical and textbook manner with no student voice in evidence then there is only evidence of some engagement.

Research type explorations do not allow much scope for personal engagement. There are few if any opportunities for the student to ask questions and show creativity in the application of the mathematics.

Students who were awarded higher levels here used their interest to design appropriate experiments and/or surveys that yielded relevant data or measurements. These students justified the design of the questions as well as the relevant sampling process in the context of the aim. This was followed by appropriate and creative mathematical analysis.

It was noted that more students were able to show significant engagement by exploring different, relevant perspectives and approaches to achieve the aim. Successful students who completed modelling-based explorations acknowledged the limitations of their techniques, introduced refinements as well as researched and found other relevant functions applicable to the context of the aim. Less successful students mechanically selected different functions with no consideration of the phenomenon being modelled, showing little to no evidence of personal engagement or creative use of mathematics.

Unfortunately, there were many centres where the explorations appeared to be following a given structure/template. These types of explorations were mechanical and directed, leaving no room for personal engagement and creativity.

Criterion D

Achievement in this criterion did not show much improvement this session. Although correct and relevant mathematical processes were seen, students continue to struggle with reviewing, analysing methods and evaluating the results in the context of the type of data and/or aim.

More attempts to achieve the higher levels were seen, however this was sometimes done through lengthy discussion of source and data validity, suggesting improvements which were often unrealistic to complete by an SL IBDP student with limited mathematical knowledge. Personal opinions not related to the mathematics/results were often stated. Students also tried to give highly in-depth reflection, by considering every possible assumption and source error that they could think of, resulting in long paragraphs on areas not related to the aim.

In statistics-based explorations students should reflect on the validity and appropriateness of processes used in the context of their exploration and the nature of the collected data. Simple explorations such as those where the aim is to find the correlation between two variables seldom give students and opportunity to state any meaningful reflections. In this context making statements such as “more data needed” without evaluating the strength and limitations of the initial sampling would be seen as limited reflection.

When evaluating models in a modelling-based exploration a student could for example reflect on how mathematically appropriate the chosen model would be for the phenomenon, what the appropriate domain and range would be and why.

Once again it should be emphasized that reflection should be seen throughout and be used to guide the progress of the exploration rather than just a paragraph at the end listing generic limitations and further research/extensions. Students should be guided to reflect on the results of each section before moving onto the next.

Criterion E

Students must clearly demonstrate understanding of the relevant and appropriate mathematics used to achieve the aim to be awarded level 3 and above in this criterion. It is thus important to note that obtaining the correct answer to a mathematical process does not necessarily demonstrate understanding.

Understanding is shown through ensuring the relevance of the mathematics used to achieve the aim as well as the explanations and interpretation of the results and processes used.

Top scoring statistical explorations demonstrated evidence that the students considered the type of data collected, the source and validity of the data, the sampling process, the most appropriate processes to apply in context of the aim and the assumptions of the statistical processes intended for use. This implies that the type of sampling was considered in context and the process was described. These students also considered the assumptions required for the statistical tests/processes they intended to use as well as whether these tests/processes are applicable to the context and aim.

In less successful explorations students seemed to select and apply as many statistical processes as possible without any justification or consideration of the context and aim. Thus, performing univariate data analysis without clearly stating how this would be relevant to the aim of correlation, removing univariate outliers without considering if these would also be (genuine) bivariate outliers, calculating the PMCC value before looking at the scatterplot, finding the linear regression line with a low PMCC value, applying Spearman's rank without clear mathematical justification and performing χ^2 -test for independence without explaining the relevance to correlation would indicate limited understanding of the mathematics used irrespective of achieving the correct numerical values. These types of explorations were very mechanical in nature with little evidence of understanding.

It is important that students be made aware that correlation does not imply causation/impact/effect/influence and thus should not conclude with statements such as "this shows/proves that A impacts/affects B negatively/positively" unless they have clearly shown causality.

Some common errors noted in the application of the χ^2-test for independence and the GOF test:

- hypotheses stated incorrectly and not using the appropriate terminologies
- stating the significance level *after* calculating the *p*-value or statistical value
- presenting the observed matrix but not the expected matrix in the body of the exploration
- not recognizing that expected frequencies less than 5 or df = 1 requires further action
- blindly applying Yates Correction when expected frequencies are less than 5 or df = 1 without explaining why this is required
- not considering regrouping categories when expected frequencies are less than 5
- not justifying the bin sizes if original data was continuous
- mistakenly stating that the test for independence would "prove" or "disprove" correlation

The modelling-based explorations showed improvement as there was evidence to suggest that students have been made aware of the modelling process as stated in section 2.6 of the Mathematics A&I SL syllabus. The stronger students considered the type of data and the properties of the phenomenon to be modelled in the context of the properties of the proposed model functions, giving sound justification for the choice of model. Less successful students overlooked the data trends, merely stated a list of possible functions (some even before presenting the data) and using a guess and check process together with the coefficient of determination to choose a model, often using polynomial functions of very high degrees with little to no evidence of understanding.

Explorations based on applying Voronoi diagrams were generally well done. However, in some cases the students could follow the algorithm but could not show understanding of the properties and the appropriate application of the results in the context of the aim.

Recommendations and guidance for the teaching of future students

In general, teachers should guide students to explicitly state an achievable aim in the introduction. This aim should be clear and specific and within the student's mathematical grasp.

Students should be encouraged to explore a variety of topics from the syllabus rather than guiding them in a single direction. There are many topics in the syllabus other than statistical analysis. This would help students to engage more in choosing a topic of real personal interest.

Students would benefit from more explicit modelling of what meaningful and critical reflection would look like. This could be done by regularly building in opportunities in class where students could write about what they have learnt, what they could do to improve their work, what the limitations were of their approaches in solving problems.

It is important that students are exposed to the criteria and the level descriptors as early as possible in the course. Students could then use these to grade sample explorations which are available in the TSM. Teachers could then extract the common problems mentioned in this and all the previous exam reports and initiate class wide discussions on how to avoid these errors.

As stated in the subject guide:

"Teachers and students must discuss the exploration. Students should be encouraged to initiate discussions with the teacher to obtain advice and information, and students must not be penalized for seeking guidance**. As part of the learning process, teachers should read and give advice to students on one draft of the work. The teacher should provide oral or written advice on how the work could be improved, but not edit the draft.** The next version handed to the teacher must be the final version for submission".

If this process is completed accurately, then many of the noted errors and misconceptions could be rectified for the final version of the exploration.

Further comments

There were far too many explorations where there were no teacher annotations, nor any comments noted on the exploration itself. The IB Teacher Support Material states that at the end of the exploration process teacher responsibilities include verifying the accuracy of all calculations and indicating on the exploration where mistakes have been made as well as annotating the exploration appropriately to indicate where the teacher's achievement levels have been awarded. Merely making the odd tick with a criterion letter is generally not helpful without further elaboration.

Teacher comments justifying the levels awarded should be more than a rewording of the criteria descriptors. Comments provided by the teacher justifying the levels awarded should be specific to each student's work.

The exploration should be double spaced, font size of minimum 11 and must contain a page count which excludes the title page and the appendices. Pages should be numbered.

Teachers should double check that the exploration has been uploaded/submitted correctly. There were again too many explorations seen where all/some of the mathematical expressions were missing, and the diagrams shifted or were even upside down. If in doubt, it might be necessary for the explorations to be converted to PDF format prior to uploading to avoid the mentioned issues.

Providing a template for statistical explorations was prevalent this session and should be avoided as this usually prevents students from achieving higher levels in most criteria.

If there is more than one teacher marking explorations in the school, then internal moderation is advised to ensure consistency in the application of the criteria.

Lastly, it is very important that no personal details such as the name of the teacher/student or the name of the school is included in any of the documentation including the teacher comment page.

Standard level paper one

General comments

Unstructured questions where the students must decide on an approach and carry it through continue to be challenging. It is beneficial for students to carefully read a question, decide what is being asked, and determine what mathematics needs to be used.

Students would be wise to attempt every question and every question part, even if they are unsure how to complete the question. Knowledge and skills can often be rewarded when a suitable approach to a problem is demonstrated, even if a final answer is not reached. This cannot happen if a question is left blank.

The areas of the programme and examination which appeared difficult for the students

- Distinguishing statistical range from range of a function.
- Finding the lower bound of a measurement.
- Recognizing events that are not mutually exclusive and explaining why they are not.
- Finding the sum of a geometric series that does not begin with the first term.
- Solving exponential equations.
- Recognizing and describing the significance of an asymptote of a function.
- Distinguishing situations where the compound interest formula can be used and when a financial application on the calculator must be used.
- Identifying edges of a Voronoi diagram.
- Determining the parameters of a sinusoidal function.
- Rearranging a formula.
- Realizing that a standard deviation is unaffected by adding a constant to each value in a data set.
- Recognizing the derivative as a rate of change.
- Writing a model for a direct variation that involves a square or cube.
- Solving complex problems involving expected value and explaining fair games.
- Providing clear and effective justification in words.

The areas of the programme and examination in which students appeared well prepared

- Finding descriptive statistics (mean, median, mode, range) of a data set.
- Interpreting Venn diagrams.
- Using geometric sequence and series formulae appropriately.
- Solving basic financial problems.
- Writing an equation of a line.
- Solving simultaneous equations.
- Calculating the volume of a cube.
- Finding the derivative of a polynomial function.
- Using the Pythagorean theorem.
- Using trigonometric ratios to find the measure of an angle in a right-angled triangle.

- Appropriately using and substituting into the arc length formula.
- Using the expected value formula and interpreting the result.
- Substituting into a given equation or formula.

The strengths and weaknesses of the students in the treatment of individual questions

Question 1 Measures of central tendency; lower bounds of rounded numbers

Most students could find the mean, either using the mean formula or the graphic display calculator, and the mode. Finding the median was a little more difficult with some students neglecting to arrange the values in numerical order prior to finding the middle value. Finding the range was much tougher with students often writing an inequality rather than a single value or subtracting the first and last values listed in the table. Part (b) was a challenging question for many of the students, with 1.97 being a popular answer. Several students mistakenly calculated 2.1 as an answer from using 1.98 as the new mean height rather than Gheorghe's measured height.

Question 2 Venn diagrams and probability

Parts (a), (b), and (c) were well done. The most common error was omitting the 2 when counting the total number of students surveyed. In part (d), many students interpreted this question as asking for those who ate at one café only. Several tried to use the formula, unsuccessfully, rather than the more efficient method of counting from the Venn diagram. There were many students who did not write their answer as a probability. Part (e) had mixed results. Some students provided an adequate rationale for stating the events were not mutually exclusive. The remaining students either provided a rationale that did not support their conclusion or stated that the events were mutually exclusive. There was some confusion between mutually exclusive events and independent events.

Question 3 Geometric sequence and series

This question was very well done. Students selected the appropriate formulae and used them correctly. In part (b)(ii), there were some students who calculated the total number of cars sold in 24 months, and mistakenly presented this calculation as their answer, neglecting to subtract the number of cars sold in 2025.

Question 4 Modelling with exponential functions

Most students were successful in finding the answer to part (a). In part (b), there were a number of students who correctly substituted, and then tried to solve their equation algebraically, with little success, rather than using their graphic display calculator. For the most part, students were able to draw the graph of the function, although the precision required for the draw command was sometimes lacking. Several students had difficulty providing clear and concise mathematical reasoning, as was evident in part (d) responses, with only a few referencing the horizontal asymptote.

Question 5 Compound interest and annuities

Students who used the financial app for both parts of this question met with success in most cases, although often they would show positive signs for both PV/FV in part (a) and PV/PMT in part (b). Those students who attempted to use the compound interest formula were able to substitute in part (a), but were unsuccessful if they tried to solve algebraically instead of using the graphic display calculator. Most of these students did not realize that the compound interest formula could not be used in part (b) because of the regular monthly payments.

Question 6 Voronoi diagrams

Parts (a) and (b) were generally well done, although the answer to part (b) did not always reflect the specified rounding in the question. In part (c), only a few students demonstrated understanding of the edges of a Voronoi diagram, with many students drawing more than three edges and many drawing edges not on the perpendicular bisectors. Several students joined points A, B, and C as the edges.

Question 7 Sinusoidal models

In general, students had difficulty finding the parameters of the sinusoidal equation from the graph. Very few students identified the period of 12 in their attempt to find the b-value. In part (c), using their incorrect parameters for a and b, most students realized they should set their equation to 3, or identified $d = 3$ on the graph as a key to the solution. Some students attempted to estimate *t* from the graph, often resulting in an inaccurate answer of 6:30.

Question 8 Rearranging formulae and effect on the mean and standard deviation when changes are made to data

In part (a), rearranging the given formula to isolate *C* proved to be difficult for several students, with many incorrect processes used. It may have benefitted students to attempt the isolation in two steps rather than trying to do everything at once. Converting temperatures in (a)(ii) and (b)(i) proved to be very easy for nearly all students, since only a simple substitution was required. Very few students realized that the addition of 32 would have no effect on the standard deviation, with the most common error being a conversion of the standard deviation by substituting into the given formula. The occasional correct solution often showed an alternate method of converting Celsius temperatures that were one standard deviation apart to degrees Fahrenheit, and then subtracting these answers.

Question 9 Derivative of a polynomial function

Part (a) was well done. Unfortunately, there were a few students who found the volume in part (ii), but did not include the units or wrote incorrect units. The majority of students were able to find the derivative of the function, with the odd error here and there. In part (c), several students did not realize that the derivative was to be used to find the rate of change, and rather substituted into the original volume function. In part (d), many of the rationales provided were adequate; however, some students referred to the sugar cube not being fully submerged or attempted to describe why a different sugar cube would dissolve at a different rate.

Question 10 Direct variation

Students had a great deal of trouble with both parts of this question. In part (a), many students wrote a linear direct variation equation (e.g. $m = 80h$), ignoring the cube. Others wrote an expression for m that did not include a constant (e.g. $m = h^3$). In part (b), many students simply divided the two masses in an attempt to find *k*.

Question 11 Arc length and right-angled triangle trigonometry

Most students could find the radius of the circle using Pythagorean theorem. Several students assumed that $\mathrm{B\hat{O}A} = \mathrm{B\hat{O}C} = 45°$, and worked through the problem with that error. Usually, those students who could find the correct measure of $B\hat{O}A$ or $B\hat{O}C$, could successfully substitute into the arc length formula and derive the correct answer.

Question 12 applications of expected value

Since part (a) of the question asked for expected value, several students knew to use the expected value formula, used it correctly, and were able to do an accurate interpretation of their answer. There were quite a few students, however, who identified the probability of winning (40%) as the expected value or calculated only the product for winning as the expected value (5×0.40). Part (b) proved to be very difficult for students, with many identifying either 12 (the winning dollar amount when the game becomes fair) or 7 (number of price increases to make the game fair) as their answer, rather than continuing on to find the number of games. Most students still attempted to provide an explanation for part (ii), but often referred to the probability of winning vs losing rather than to the expected value prior to the game being fair.

Recommendations and guidance for the teaching of future students

- Recognition of command terms and the expectation of each (e.g. “draw” vs. “sketch”).
- When using a financial app, it is important to pay attention to positive and negative signs for PV, PMT, and FV, and to show all calculator entries.
- When solving a direct or inverse variation problem, begin by writing a generic variation equation that includes a constant (e.g. $m = kh^3$ or $y = k/x$), and then progress from there.
- Practise explaining, justifying responses, and providing rationales in words.
- Practise with, and exposure to, unstructured problems so that students become used to developing their own problem-solving strategies without prompts.
- Include units for any question where it is appropriate to provide clarity and correctness.
- Identifying those problems where using the functionality of a graphic display calculator is preferable and more successful than an algebraic approach.
- Attempt each question part regardless of where it occurs in the paper rather than leaving questions completely blank.
- Avoid crossing out working if it is not replaced with other working. Crossed-out working is not marked and yet is occasionally correct.
- Work should be neat and responses labelled with the question number. On occasion, responses are illegible and/or it is difficult to identify which question part is being answered.
- Responses are to be written in the answer boxes. Several students wrote their answers by the questions or on extra sheets, leaving the answer boxes blank. Likewise, answers in the answer box should not be duplicated on extra sheets.
- All working should be shown. If no working is shown, students will earn no marks if the answer is incorrect.
- Awareness that if students write multiple solutions for a question, only the first solution presented is accepted and marked.
- Pay attention to any specified rounding in a question. If there is no rounding specified, final answers should be written exactly or to three significant figures as stated in the examination instructions.
- Rounding prematurely while working through a question can lead to incorrect answers; rounding should be done only when a final answer is reached.
- Ensure that all topics in the syllabus are covered, including a review of prior learning topics.

- It would be helpful to expose students to previous examination questions so that they are familiar with the syntax, terminology, and style of questions that may be asked.

Standard level paper two

General comments

A pleasing number of students demonstrated a good level of understanding necessary to successfully complete the paper. It was encouraging to see most parts attempted, though engagement with the questions seemed to trail off at the tail end of the paper. Time constraints did not appear to be a significant factor. Students effectively utilized their GDC and provided well-supported answers. Some marks were lost due to accuracy issues, with a common error being the interpretation of the first zero after a decimal place as a significant figure.

The areas of the programme and examination which appeared difficult for the students

- Stating assumptions about the distributions for a *t*-test.
- Recognizing a binomial event.
- Writing down a piecewise function for a restricted domain.
- Answering 'show that' questions.
- Performing a statistical test with little scaffolding or support.
- Finding the integral for a given boundary condition.
- Forming three equations to satisfy design constraints.
- Evaluating a mathematical model in the context of the problem.

The areas of the programme and examination in which students appeared well prepared

- Linear regression.
- Performing a *t*-test and interpreting the results.
- Finding probabilities for a normal distribution.
- Substitution into a model.
- Differentiating a function.
- Completing a tree diagram.
- Compound probabilities.
- Finding the expected value.
- Using a GDC to find the area under a curve.

The strengths and weaknesses of the students in the treatment of individual questions

Question 1

Students should be mindful of command terms and the number of marks allocated. Students chose inefficient routes such as writing down the ranks to find Spearman's rank correlation coefficient or erroneously finding the coefficient of determination. In this question, it was intended that a student could write down r_s through inspection of the scatterplot.

Students were usually able to find the Pearson's product-moment correlation coefficient, though a few omitted the negative sign. It is apparent that some students missed the demand of the question. This was evident when those with a positive value of PMCC would sometimes describe the correlation coefficient from the scatterplot rather than their value of r, as directed in the question.

The level of accuracy in student answers is concerning. Teachers need to constantly remind their students that any answer not given exactly or to 3 significant figures is considered incorrect. This was particularly an issue in (c)(i) when writing down the value of the parameter 'a', with students perhaps interpreting the leading zero after the decimal point as a significant figure.

Interpreting the value of b in context was problematic for some. Common errors included imprecise answers such as the price 'in the city', rather than the price in the centre of the city. Another common error was not identifying this as a price.

Using the regression equation for the purpose of prediction was generally answered without error, though many failed to recognize the answer in millions of dollars. Students should be encouraged to recognize the reasonableness of the answer in the context of the problem. The price of an apartment is unlikely to be $1.25. An unrealistic answer should alert the student to a possible error.

Stating the reasons to justify the validity of an estimate for a regression equation is a familiar task and should be no surprise to students who have practiced past examination questions. Even so, many offered answers such as 'the trendline follows the data' or 'the data point is very close to the regression line.' While these answers have some merit, they accrued no marks.

Students appeared more comfortable with the t-test and were able to write down the alternative hypothesis. Students should still be reminded that this is a test to see if the sample data provides sufficient evidence to reject the null hypothesis that the population means are equal. It is a test of inference about the populations and not about the samples. A number of students incorrectly chose the unpooled option in their GDC or performed a one-tail test, both of which resulted in errors. In part (h), it was surprising to see some students re-state the assumption 'the variances are equal' when this information was given earlier in the question.

Question 2

When finding $\mathrm{P}(T < 64)$, some did not recognize the numerical limitations of their GDC, stating their answer as 0.499999… or 0.5000000001.

The current generation of students are digital natives and so are more technologically literate. Most students were able to perform routine tasks in their GDC. In part (b), $\mathrm{P}(44 < T < 64)$ was correct for the majority of students. Errors in this part were usually attributed to an answer given to two significant figures or interpretating the variable as discrete rather than continuous.

Most students drew a normal curve, though some poor sketches were seen. Common breaches in tolerance included crude sketches with no change of curvature towards the extreme values, poor symmetry, or not reflecting asymptotic properties of the curve. Students should support their diagram with labelling of the mean and k. Most shaded the left-hand tail. It was encouraging to see many students supporting their work with an attempt at working for the inverse function when finding k in part (c)(ii).

Many did not recognize the binomial event in part (d). Those who did usually were rewarded with full marks.

It was pleasing to see a few students find the correct expression for the amount charged to deliver a package for $x > 1$. The majority, however, did not consider the domain, incorrectly writing their expression as $2x + 4.5$. On a positive note, students with an error in part (e) usually went on to score follow-through marks in the subsequent parts of the question.

Question 3

Most students picked up early marks finding the maximum price in part (a) and the number of smoothies sold and total income in part (b).

The vast majority of students failed to make any headway with part (c), usually substituting in $x = 2$ into the expression for P. The substituting of known values could at best satisfy a command term 'Verify' but is not generally appropriate in a 'Show-that' question. A few students had modest success by correctly multiplying the cost of each smoothie by the number of smoothies sold per day. Finding the total revenue per day proved more problematic, with only the best student responses accruing full marks.

It was pleasing to see many students attempt to express P with negative indices in readiness for the power rule. Relative to past exams, more students were able to find the correct derivative. However, it is clear that some students need more practice differentiating negative powers.

There were a surprising number of incorrect answers in parts (c)(iii) and (iv). Although follow-through marks were awarded, some students lost marks for answers that were either negative or unrealistic.

Question 4

On the whole, finding the values of a, b, and c in the tree diagram was well done. Students successfully found the probability of both switches failing in part (b).

"Show that" questions are often a source of difficulty for students. However, it was pleasing to see many students correctly prove and write down the final statement.

Multistep solutions with less scaffolding are usually vexing for students, and in part (d) many had difficulty setting out their work in a logical and coherent manner. In a χ^2 goodness of fit test, even when not prompted to, students should be encouraged to support their expected frequencies with calculations and write down the degrees of freedom for the test. Some gave their p-value to one significant figure. Sometimes the incorrect conclusion was given while providing the correct reasoning. However, the majority of students wrote a conclusion consistent with their p-value and were able to justify their reasoning.

Question 5

Students were able to substitute correctly into the differential equation in part (a). At this stage in the paper, some capable students started to run out of steam. Even so, it was encouraging to see many make a reasonable attempt at integration, finding two correct terms. A common error was omitting the constant of integration, which made it impossible to use the initial boundary condition.

The impact of the omission of the constant of integration in part (b) cascaded into parts (c) and (d), with students finding a negative area and a negative volume. This should have alerted students to a possible error in their expression for $h(x)$. Other students did not appear to recognize that Volume = cross-sectional area x length, so they tried to find the volume by using length x base x height.

Stronger students were able to write down three equations in terms of a, b, and c. Some lost one mark for not simplifying their equations. The general marking rules state that students are advised to give final answers using good mathematical form. In general, for a final mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified.

Those who used a GDC in part (f) were usually able to find their correct values of a, b, and c. Students who adopted an analytic approach fared less well.

Part (g) proved to be a true discriminator for even the strongest students. Few were able to make even one correct comparison of heights or gradients. Teachers should guide their students that preliminary answers might have importance later in the question, and therefore should try to understand the significance of their results.

Recommendations and guidance for the teaching of future students

- Emphasize a conceptual understanding.
- Time management. On average, one minute should be allocated per mark.
- Ensure prior learning topics are addressed in the course.
- Students should use a GDC calculator that can support all areas of the course.
- Students should show and use unrounded answers. Unless otherwise directed, students should not round their intermediate answers within a question part.
- Final answers should be expressed exact or to at least three significant figure accuracy, as per the front cover rubric of the examination.
- The given information in a question is expected to be used in subsequent parts.
- Be conscious of the meaning and significance of preliminary results in a question.
- “Show-that” commands assess communication, also function as signposts to re-enter a question.
- Include units in the answer as necessary.
- Read each question carefully.
- Reinforce analytic mathematical techniques.
- Sketch a diagram to illustrate the given information, where appropriate.
- Be familiar with the command terms and the information in the formula booklet. For example, if the command term is ‘Write down’, then a lengthy explanation or multi-step solution is not required.
- Where possible, support your answers with relevant working.
- Students should know and use the correct terminology for each area of the course.
- Ensure students can use the GDC efficiently.
- Practice contextualized questions.
- Write all responses in the answer booklet, not in the question booklet. The question booklet is not shared with the examiner.
- Mistakes should be crossed out rather than over-written, to improve the clarity of the response.
- Consider the reasonableness of an answer. Although marking instructions allow full marks in subsequent parts, following an error, full marks may not be awarded for unrealistic answers or those which contradict the given constraint in a question.
- Process and evaluate the information in a question before commencing calculations.
- Students should know and understand the underlying attributes of a distribution.
- Do not infer information from a diagram.